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LETTER TO THE EDITOR

Effective vertex of the holon–holon interaction in the t–J model

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Abstract. Within the slave-boson approach to the t–J model of high-T_c superconductivity, the vertex of the spinon-mediated holon–holon interaction is exploited in detail to find out the critical temperature of holon pairing.

The high-T_c superconductivity of copper oxide materials can be discussed as a two-dimensional (2D) pair condensation of bosonic holons [1, 2] mediated by paired fermionic spinons [3]. The t–J model in the slave-boson version [3–6] is subjected to a mean-field (MF) approximation for the spinons. Fluctuations are included [3, 6] in this scheme via the holon–spinon interaction leading to an effective holon–holon interaction (attraction) in the perturbative parameter range $t < J$, whereby the origin of the t–J model from the Hubbard model is ignored. The resulting generalized MF equations for coupled spinon (s) and holon (h) order parameters are:

(i) Spinon system

$$\Delta_k^{\mathrm{s}} = \frac{J}{N}\sum_{k'} \eta_{k-k'} \Delta_{k'}^{\mathrm{s}} \Theta_{k'}^{\mathrm{s}}(T) \tag{1a}$$

$$p_k = -\frac{1}{N}\sum_{k'} \eta_{k-k'} \varepsilon_{k'}^{\mathrm{s}} \Theta_{k'}^{\mathrm{s}}(T) \tag{1b}$$

$$\frac{\delta}{2} = \frac{1}{N}\sum_{k} \varepsilon_k^{\mathrm{s}} \Theta_k^{\mathrm{s}}(T) \tag{1c}$$

(ii) Holon system

$$\Delta_k^{\mathrm{h}} = -\frac{1}{N}\sum_{k'} U_{kk'} \Delta_{k'}^{\mathrm{h}} \Theta_{k'}^{\mathrm{h}}(T) \tag{1d}$$

$$q_k = -\frac{1}{N}\sum_{k'} \eta_{k-k'} \varepsilon_{k'}^{\mathrm{h}} \Theta_{k'}^{\mathrm{h}}(T) \tag{1e}$$

$$\delta = \frac{1}{N}\sum_{k} \varepsilon_k^{\mathrm{h}} \Theta_k^{\mathrm{h}}(T) - \tfrac{1}{2} \tag{1f}$$

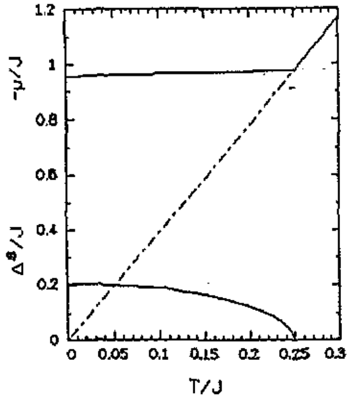

Figure 1. The dependence of the spinon-pairing amplitude, Δ^s, and the chemical potential, μ, of the separate spinon system on temperature, T, in units of the parameter J at a doping concentration of $\delta = 0.05$. The dash-dot curve refers to the chemical potential in the unpaired case.

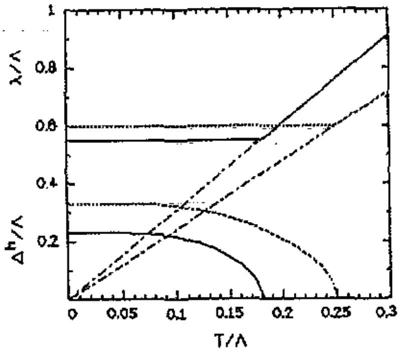

Figure 2. The dependence of the holon-pairing amplitude, Δ^h, and the chemical potential, λ, of the separate holon system on temperature, T, in units of the parameter Λ at a doping concentration of $\delta = 0.05$ (solid curve) and $\delta = 0.10$ (dashed curve). For the dash-dot curves, see figure 1.

with $\Theta^s_k(T) = (1/2E^s_k)\tanh(E^s_k/2T)$ and $\Theta^h_k(T) = (1/2E^h_k)\coth(E^h_k/2T)$. The vertex of the holon–holon interaction in (1*d*) is found to be

$$U_{kk'} = \frac{t^2}{N}\sum_q F_{kk'q}\frac{E^s_{q+k} + E^s_{q+k'}}{(\varepsilon^h_{k'} - \varepsilon^h_k)^2 - (E^s_{q+k} + E^s_{q+k'})^2}(1 - f^s_{q+k} - f^s_{q+k'}) \tag{2}$$

where

$$F_{kk'q} = (1/4E^s_{q+k}E^s_{q+k'})[\eta^2_q\Delta^s_{q+k}(\Delta^s_{q+k'})^* + \eta^2_{q+k+k'}\Delta^s_{q+k'}(\Delta^s_{q+k})^* + 2\eta_q\eta_{q+k+k'}(E^s_{q+k}E^s_{q+k'} - \varepsilon^s_{q+k}\varepsilon^s_{q+k'})].$$

$\Delta^{s(h)}_k$ and $p_k(q_k)$ denote the spinon (holon) pairing and hopping amplitudes (for definitions see reference [6]; δ is the holon (doping) concentration. The excitation spectrum $E^{s(h)}_k = +\sqrt{(\varepsilon^{s(h)}_k)^2 \pm |\Delta^{s(h)}_k|^2}$ involves the spinon (holon) energy dispersion,

$$\varepsilon^s_k = \mu - J(1-\delta) - (tq_k + Jp_k/2)(\varepsilon^h_k = \lambda - 2tp_k)$$

with $\mu(\lambda)$ as the chemical potential. The structure factor is $\eta_k = 2(\cos k_x + \cos k_y)$ in units of the lattice constant (2D square lattice); f^s_k is the Fermi function, $f^s_k = (\frac{1}{2}) - E^s_k\Theta^s_k(T)$. N and T mean the number of lattice sites and the temperature. In deducing the MF equations (1), various symmetries of Δ^s_k and $p_k(q_k)$ can be taken into account. The symmetry of Δ^h_k is determined by $U_{kk'}$. At $U_{kk'} \equiv 0$, the MF equations (1) agree with those in [5], and at s-wave symmetric Δ^s_k, p_k and q_k with those in [6].

To study the explicit coupling of the MF equations (1) via $U_{kk'}$, first the two subsystems are investigated in the low doping region. The hopping amplitudes, p_k and q_k, are assumed to be negligibly small. Consequently, there is no implicit coupling through $\varepsilon^{s,h}_k$, and the fermionic spinon system (1*a*–*c*) can be treated separately.

(i) Spinon system (1*a*–*c*). In analogy to estimations in [5], the RVB order parameter Δ_k tends to zero at the critical temperature, $T^s_c = T^{RVB} = (J\delta/2)/\ln((1+\delta)/(1-\delta))$. Assuming s-wave symmetry, the doping dependence of $\Delta^s_k = \Delta^s\eta_k$ at zero temperature is given approximately by $\Delta^s(T=0) = (2J/\pi^2)\sqrt{1-\delta^2}$. Note that at zero doping

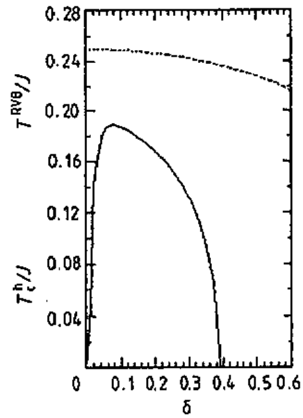

Figure 3. The critical holon temperature T_c^h in the coupled holon–spinon system (solid curve) and the RVB temperature, T^{RVB}, of the separate spinon system (dashed curve) against the doping concentration, δ, for $t/J = 0.52$.

$T^{RVB}(\Delta^s(T=0))$ has the maximum value of $J/4(2J/\pi^2)$. In figure 1 the temperature dependence of Δ^s and of μ is plotted ($\delta = 0.05$). $\Delta^s(T)$ shows a BCS-like behaviour.

(ii) Holon system (1*d*–*f*). To separate the holon system, $U_{kk'}$ is replaced by a constant attractive pseudopotential $-\Lambda(\Lambda > 0)$; $\Delta_k^h \equiv \Delta^h$ also becomes k independent. One then obtains MF equations which refer to those in [2]. The critical temperature, defined by $\Delta^h \to 0$, is now given by $T_c^h = \Lambda(1/2+\delta)/\ln((1+\delta)/\delta)$, and the zero temperature doping dependence of Δ^h is found to be $\Delta^h(T=0) = \Lambda\sqrt{\delta(\delta+1)}$. These equations are identical with those in the case of $n_0 = \delta$ in [2]. In figure 2 the temperature dependence of Δ^h and of λ is represented for $\delta = 0.05$ and 0.10; $\Delta^h(T)$ also shows a BCS-like behaviour. In contrast to $\Delta^s(T)$ in figure 1, $\Delta^h(T)$ now depends sensitively on δ.

(iii) Coupled systems (1*a*–*f*). The coupling between the two subsystems in (1) is mainly caused by the complicated vertex $U_{kk'}$ in (2). Assuming an s-wave symmetric paired-holon state, the holon pair condensation is favoured at $k \approx 0$ [3]. The temperature- and doping-dependent diagonal term of $U_{kk'}$ in (2) at $k = k' \approx 0$, called $-\Lambda_{T,\delta}(\Lambda_{T,\delta} \geqslant 0)$, now plays the role of the pseudopotential, $-\Lambda$. The holon–holon interaction is determined by the spinon state, so that

$$\Lambda_{T,\delta} = \frac{t^2}{2N}\sum_q \eta_q^2 \frac{|\Delta_q^s|^2}{(E_q^s)^3}\tanh\left(\frac{E_q^s}{2T}\right). \tag{3}$$

Obviously, $\Lambda_{T,\delta}$ vanishes in the case of unpaired spinons ($\Delta_k^s \equiv 0$). An s-wave symmetric paired-spinon state ($\Delta_q^s = \Delta^s\eta_q$) and a gapless spinon excitation spectrum ($E_q^s = E^s|\eta_q|$) [7] yield

$$\Lambda_{T,\delta} = 2g_0(\Delta^s/E^s)^2 \tag{4}$$

where $g_0 = 2t^2/J$. Using the approximations $p_k \approx 0$ and $q_k \approx \delta\eta_k$, one gets $E^s = +\sqrt{t^2\delta^2 + (\Delta^s)^2}$ [7]. The dimensionless quantity $(\Delta^s/E^s)^2$ in (4) lies between 0 and 1 in dependence on T and δ. So the strength of $\Lambda_{T,\delta}$ is scaled by the strength of the coupling constant, g_0 (see [8]). The *t–J* model in the small (J/t) limit corresponds to the Hubbard model in the large (U/t) limit, where $J = (4t^2/U)$ [9]. In that strong-coupling limit, $\Lambda_{T,\delta}$ grows indefinitely with $(4t^2/J) = U$ as pointed out in references [10, 11]. This unphysical fact gives rise to a renormalization of $g_0 \sim t^2/J(\sim U)$ to $g_0 \sim J\ (\sim t^2/U)$ [8]. A suitable renormalization procedure due to vertex corrections is still outstanding. For that reason, and in expectation of the decrease of the vertex corrections at $t < J$ [10], the ratio of the parameters is chosen here as $(t/J) = 0.52$. From equation (4) T_c^h is evaluated numerically as a function of δ in the coupled-MF system (1). The curve is shown in figure 3. T_c^h is restricted to the doping range $0, \ldots, \delta, \ldots, 0.39$ and can be identified with the superconducting transition temperature. To illustrate the requirement of a background of paired spinons, T^{RVB} for the separate spinon system is also plotted in figure 3. The curvature of $T_c^h(\delta)$ and $T^{RVB}(\delta)$ shows a remarkable similarity with that of $T_s(\delta)$ and $T_m(\delta)$ of calculations in the p–d model in [12].

Finally one can say that the 2D pairing mechanism for charged holons caused by the exchange of uncharged spinons is formally described by the effective vertex of the holon–holon interaction. The critical temperatures are defined based on pairing order parameters. The use of the effective vertex leads to a link between the strong-coupling and the weak-coupling limits [10]. This problem seems to be independent of the scheme (either slave-boson [3–6] or slave-fermion [6, 13] version).

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